

Boolean laws

$P \cap Q = Q \cap P$
 -- commutativity
 $P \cap (Q \Delta R) \Delta P \cap Q = P \cap R$
 $P \cap Q \cap R = P \cap (Q \cap R)$
 -- associativity
 $P \Delta Q \Delta R = P \Delta (Q \Delta R)$
 -- associativity
 $_ \cap \emptyset = \emptyset$
 -- multiplicative annihilator
 $P \cap P = P$
 -- idempotency
 $P \cap (P \cap Q) = P \cap Q$
 $P \cap Q = P \& Q \cap P = Q \rightarrow Q = P$
 $P \cap Q = Q \& Q \cap R = Q \rightarrow P \cap R = P$
 $P \Delta Q = Q \Delta P$
 -- commutativity
 $P \Delta (Q \Delta R) = Q \Delta (P \Delta R)$
 -- permuted associativity
 $\emptyset \Delta P = P$
 -- additive unit
 $P \Delta P = \emptyset$
 $P \Delta (P \Delta Q) = Q$
 $P \cap (Q \Delta R) = P \cap Q \Delta P \cap R$
 -- distributivity

$P \setminus Q = P \Delta P \cap Q$
 -- : this could be taken as a definition
 $P \subseteq Q \leftrightarrow P \cap Q = P$
 -- : this could be taken as a definition
 $P \subseteq Q \rightarrow Q \setminus (Q \setminus P) = P$
 $P \subseteq Q \& R \subseteq Q \rightarrow (Q \setminus P) \cap R = R \Delta P \cap R$
 $P \subseteq Q \& R \subseteq Q \rightarrow (Q \setminus P) \Delta R = Q \setminus P \Delta R$
 $(_ \setminus P) \cap P = \emptyset$
 $P \subseteq Q \rightarrow P \Delta (Q \setminus P) = Q$
 $P \subseteq Q \& R \subseteq Q \rightarrow (P \cap R = P \leftrightarrow P \cap (Q \setminus R) = \emptyset)$
 $P \subseteq Q \& R \subseteq Q \rightarrow P \Delta R = Q \setminus (Q \setminus (Q \setminus P) \cap R) \cap (Q \setminus P \cap (Q \setminus R))$
 $P \subseteq Q \& R \subseteq Q \rightarrow Q \setminus P \Delta R = (Q \setminus (Q \setminus P) \cap R) \cap (Q \setminus P \cap (Q \setminus R))$
 $P \cup Q = P \Delta Q \Delta P \cap Q$
 -- : this could be taken as a definition
 $P \cup Q \cup R = P \cup (Q \cup R)$
 -- associativity
 $P \cup Q = Q \cup P$
 -- commutativity
 $\emptyset \cup P = P$
 -- unit element
 $P \cup P = P$
 -- idempotency
 $P \cap (Q \cap (P \cup _)) = P \cap Q$
 $P \cup P \cap _ = P$
 $P \subseteq Q \& R \subseteq Q \rightarrow (Q \setminus P) \cap R \cup P \cap R = R$
 $P \subseteq Q \& R \subseteq Q \rightarrow Q \setminus (P \cup R) = (Q \setminus P) \cap (Q \setminus R)$
 $P \cup (P \cup Q) = P \cup Q$
 $P \cup (Q \cup R) = Q \cup (P \cup R)$
 $(P \cup Q) \cap (P \cup R) = P \cup Q \cap R$
 $P \subseteq Q \& R \subseteq Q \& S \subseteq Q \rightarrow Q \setminus P \cup (R \cup P \cap S) = Q \setminus P \cup (R \cup S)$
 $P \subseteq Q \& R \subseteq Q \& S \subseteq Q \rightarrow P \cup R \cup (Q \setminus P) \cap S = P \cup (R \cup S)$
 $P \cup _ = \emptyset \rightarrow P = \emptyset$
 $P \subseteq Q \& R \subseteq Q \rightarrow P \Delta R = P \cap (Q \setminus R) \cup (Q \setminus P) \cap R$
 $P \subseteq Q \& R \subseteq Q \rightarrow (P \cup R) \cap (Q \setminus P \cap R) = P \cap (Q \setminus R) \cup (Q \setminus P) \cap R$
 $P \subseteq Q \& R \subseteq Q \rightarrow P \Delta R = P \cap (Q \setminus R) \cup (Q \setminus P) \cap R$